# Using Representations and Modeling to Develop Conceptual Understanding in Math


**New York City Mathematics Project**
**25th Annual Educator's Conference**
**Lehman College**
**Bronx, NY**
**May 21, 2016**

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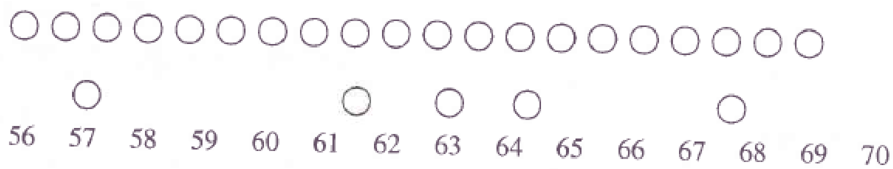
## Directions for the Human Box and Whisker Plot

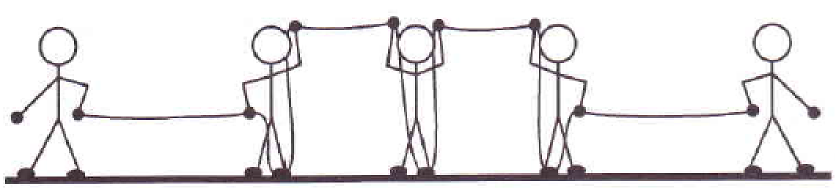
### Human Box Plot Activity

Depending on the size of your class, the Human Box Plot Activity is best done outside or in a gym or hallway.

- Have the students arrange themselves in a line according to their heights. As they are arranging themselves, place index cards or mark the floor at 1-foot intervals with the numbers 56 to 75 to represent the heights of the students in inches. You may need to adjust the minimum or maximum to fit your class situation. These numbers should be in a line parallel to the student line a few feet away.
- Ask the students to find the median person and have that person stand in front of the number that is his or her height. (If the median is between two people, choose one of them—perhaps by a coin toss—to stand at the mean of their two heights.)
- Ask the students to find the median student of each half of the split group, and have each of these students stand in front of his or her height. (See the note above if the median is between two students.)
- Have the first and last person in the line (the minimum and maximum values or extremes) stand in front of their heights.
- You should have five people standing in a line away from the group, as in the diagram below.




- Starting at one end, pass the clothesline from person to person among the five students who are standing on the numbers, having them hold the line as shown in the diagram below.



Guide a discussion on how this graph was formed. Upon returning to the classroom, the students can write their height data on the board and re-create the box plot on paper.

Extension for this activity: After the students have modeled the box plot and you have discussed all the elements, have the students form a histogram by letting everyone form a line behind their height. After the elements of a histogram have been discussed, change the interval by picking up every other index card or having the students stand on multiples of two. This is a dynamic demonstration of the meaning of the interval on a histogram.

From Advanced Algebra Through Data Exploration, Key Curriculum Press, 1998.

ONE MILLION DOLLAR PROJECT Name:

Period: Date:

## Imagine this situation:

One night, as you are about to go to bed, the telephone rings and a friend offers you a chance to become a millionaire. He tells you that he won $2 million in a contest. The money was sent to him in two suitcases, each containing $1 million in one-dollar bills. He will give you one suitcase of money if your mom or dad will drive him to the airport to pick it up. Could your friend be telling you the truth? Can he make you a millionaire?

## Some questions to consider if you want to help your friend and be rich!

1. Can $1 million in one-dollar bills fit in a standard-sized suitcase, a suitcase that you can carry? **Explain.**

2. Could you lift the suitcase if it contained $1 million in one-dollar bills? Estimate its weight.

3. Calculate the weight of the suitcase.

4. If not, what is the smallest denomination of bills you could use to fit the money in a suitcase. **Keep in mind that all denominations of United States currency have the same measurements.**

## <u>Your assignment</u>

A. You will not be able to actually use a million one-dollar bills, so you must use an indirect method. The dimensions of a one-dollar bill are 6.14 inches by 2.61 inches by 0.0043 inches [If you used metric measurements, it would be 156 millimeters by 66.3 millimeters by 0.11 millimeters.]. The dimensions of the suitcase you will use to transport the money has dimensions 28 inches by 19 inches by 13 inches [If you used metric measurements, it would be 71.1 centimeters by 48.3 centimeters by 33 centimeters.]. Twenty one-dollar bills weigh approximately 0.7 ounces. Based on these measurements, answer questions 1 through 3. **Show all work necessary to justify your answer!**

B. Write a paragraph in which you tell how you made your determination, and compare the calculated weight with your guess. **Show all work necessary to justify your answer!**

C. Answer question 4. **Show all work necessary to justify your answer!**

*Your grade will be based on neatness, accuracy, clarity of explanation, and the amount of work you show. Use an appropriate unit of measure. You may need to convert to a larger unit of measure, such as pounds, tons, or kilograms! Be sure to turn it in on time! There will be a one letter grade deduction for each day that it is late! Good luck!*

ONE MILLION PENNY PROJECT Name:

Period: Date:

## How tall might a stack of one million pennies be?

A. Guess at the height and record your guess.

B. Now try to determine the height without guessing. Of course you will not be able to actually be able to use a million pennies, so you must use an indirect method. Measure the thickness of ten pennies. Based on this measurement, how tall is a stack of one million pennies?

C. Write a paragraph in which you tell how you made your determination, and compare the calculated height with your guess. Show any and all work necessary to justify your answer.

D. If the pennies were laid down next to each other in a line, how far would the one million pennies stretch? Show any and all work necessary to justify your answer.

E. Think of some related questions. Imagine a million of some other object. Indicate how high you think they would be if placed in a stack.

***Your grade will be based on neatness, accuracy, clarity of explanation, and the amount of work you show. If you did some research about the properties of pennies, be sure to list your source(s). Use an appropriate unit of measure. You may need to convert to a larger unit of measure, such as feet, meters, yards, kilometers, or miles! Be sure to turn it in on time! There will be a one letter grade deduction for each day that it is late! Good luck!***

# INVESTIGATION 2.4: **Growing Tall**

Did You Know?

Did you know that if you are considering fast growing wild trees in the United States, the Eastern Cottonwood tree is one of the fastest if not the fastest of those trees? Under the right conditions it can grow as much as 10-15 feet per year. It does not sustain that kind of growth for its entire life, but it is common for it to grow about 5 feet per year and approximately 1 inch in diameter on average during a 25 year period.

**References and more information**

**http://www.funtrivia.com/askft Question13317.html**

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Aleisha and Bernice both are growing plants. A month ago, Aleisha's plant was 60 centimeters tall; it is now 100 centimeters tall. A month ago Bernice's plant was 25 centimeters tall; it is now 50 centimeters tall. Though they are great friends, they are also very competitive with each other. Each claims that her plant is growing faster.

**A.** Provide a convincing argument that Aleisha's plant is growing faster than Bernice's. Demonstrate your argument numerically and graphically. Also show why your argument is correct by describing what will happen over several months if the established patterns would continue.

**B.** Provide a convincing argument that Bernice's plant is growing faster than Aleisha's. Demonstrate your argument numerically and graphically. Also show why your argument is correct by describing what will happen over several months if the established patterns would continue.

**C.** Summarize mathematically why both have a legitimate claim. If you had to choose which plant is growing faster, which one would you pick?

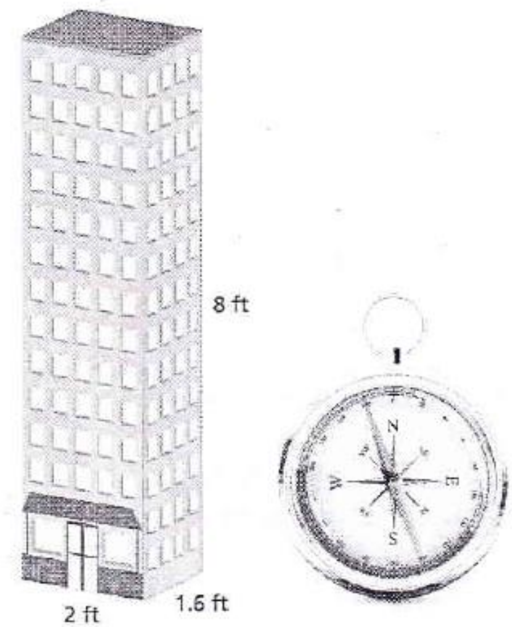
## Reference

Horton, Robert, *Fostering Mathematical Thinking in the Middle Grades with Casio Technology*, Dover, NJ: Casio American, Inc., 2013.

Name________________________________ Date__________

# CCSS 7.G.1 Architecture

An architect builds the following scale model of a 480-foot tall building.




1. What is the scale factor of the model?

2. Find the actual length of the east side of the building.

3. Find the actual length of the south side of the building.

4. What is the actual perimeter of the base of the building? How does this relate to the perimeter of the base of the model?

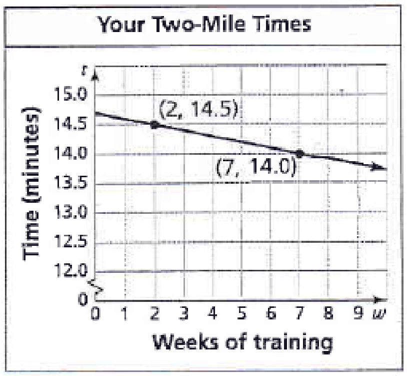
5. How much floor space is there on one floor of the actual building? How does this relate to the amount of floor space on one floor of the model?

Solomin Andrey/Shutterstock.com

Name ______________________________ Date __________

# CCSS 8.F.2 Cross Country

You and your friend are training for the cross country team. The graph shows your two-mile times $t$ (in minutes) as a function of the number of weeks $w$ you have trained. The table shows your friend's two-mile times $t$ (in minutes) as a function of the number of weeks $w$ your friend has trained.




| Your Friend's Two-Mile Times | |
|---|---|
| Weeks of training, $w$ | Time, $t$ |
| 0 | 13.75 |
| 1 | 13.70 |
| 2 | 13.65 |
| 3 | 13.60 |
| 4 | 13.55 |
| 5 | 13.50 |
| 6 | 13.45 |
| 7 | 13.40 |
| 8 | 13.35 |
| 9 | 13.30 |
| 10 | 13.25 |

1. Who had a faster time before you began training?

2. Who had a faster time after 10 weeks of training?

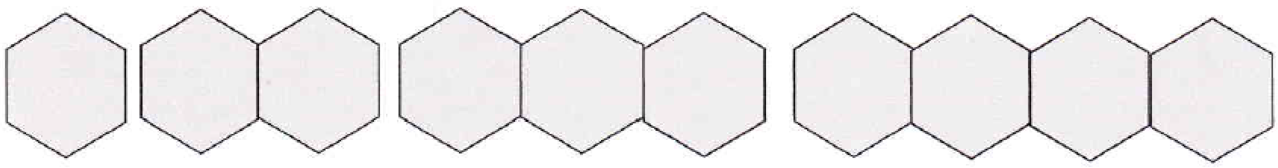
3. Whose time is decreasing faster? Justify your answer.

4. Is either function a linear function? If so, write the linear function(s).

5. If the patterns continue, after what week will you and your friend have the same two-mile time?

## Hexagon Pattern Train Task

Trains 1, 2, 3, and 4 (shown below) are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



train 1 train 2 train 3 train 4

1. Compute the perimeter for each of the first four trains.

2. Draw the fifth train and compute the perimeter of the train.

3. Make some observations that could help you describe the perimeter of larger trains.

4. Determine the perimeter of the 25th train without constructing it.

5. Write a description that could be used to compute the perimeter of any train in the pattern. Explain how you know it will always work.

6. Write an equation that could be used to compute the perimeter of any train in the pattern.

7. What is the real world meaning of rate of change?

<u>Extension:</u> How could you find the perimeter of a train that consisted of triangles? Squares? Pentagons? Can you write a general description that could be used to find the perimeter of a train of any regular polygons?

The Hexagon Train Task has been adapted from *Visual Mathematics Course 1, Lesson 16-30* published by The Math Learning Center, Salem, Oregon, 1995.